Name:______________________________ **Date:**______________

Algebra 2 CC: Zero and Negative Exponents

In general, for $x \neq 0$ and n a positive integer:

$$\frac{x^n}{x^n} = x^{n-n} = x^0$$

In order for the rule for the division of powers with like bases to be consistent with ordinary division, we must be able to show that for all $x \neq 0, x^0 = 1$.

Multiplication:	$x^n \cdot x^0 = x^{n+0} = x^n$	$x^n \cdot x^0 = x^n \cdot 1 = x^n$
Division:	$x^n \div x^0 = x^{n-0} = x^n$	$x^n \div x^0 = x^n \div 1 = x^n$
Raising to a Power:	$(x^0)^n = x^{0 \cdot n} = x^0 = 1$	$(x^0)^n = 1^n = 1$
Power of a Product:	$(xy)^0 = x^0 \cdot y^0 = 1 \cdot 1 = 1$	$(xy)^0 = 1$
Power of a Quotient:	$\left(\frac{x}{y}\right)^0 = \frac{x^0}{y^0} = \frac{1}{1} = 1$	$\left(\frac{x}{y}\right)^0 = 1$

Therefore, it is reasonable to make the following definition:

DEFINITION

If $x \neq 0, x^0 = 1$.

Multiplication:
$2^5 \cdot 2^{-3} = 2^{5+(-3)} = 2^2$
$2^5 \cdot 2^{-3} = 2^5 \cdot \frac{1}{2^3} = 2^2$

Division:
$6^2 \div 6^{-5} = 6^{2-(-5)} = 6^7$
$6^2 \div 6^{-5} = 6^2 \div \frac{1}{6^5} = 6^2 \cdot 6^5 = 6^7$

Raising to a Power:
$(3^{-4})^{-2} = 3^{-4(-2)} = 3^8$
$(3^{-4})^{-2} = \left(\frac{1}{3^4}\right)^{-2} = \frac{1}{3^{-8}} = \frac{1}{\frac{1}{3^8}} = \frac{3^8}{1} = 3^8$

Power of a Product:
$(x^2y^{-3})^4 = x^8y^{-12} = x^8 \cdot \frac{1}{y^{12}} = \frac{x^8}{y^{12}}$
$(x^2y^{-3})^4 = (x^2 \cdot \frac{1}{y^3})^4 = (x^2)^4\left(\frac{1}{y^3}\right)^4 = \frac{x^8}{y^{12}}$

Power of a Quotient:
$\left(\frac{x^3}{y^5}\right)^{-2} = \frac{x^{-6}}{y^{-10}} = \frac{\frac{1}{x^6}}{\frac{1}{y^{10}}} = \frac{y^{10}}{x^6}$
$\left(\frac{x^3}{y^5}\right)^{-2} = \frac{1}{\left(\frac{x^3}{y^5}\right)^2} = \frac{1}{\frac{x^6}{y^{10}}} = \frac{y^{10}}{x^6}$

Therefore, it is reasonable to make the following definition:

DEFINITION

If $x \neq 0, x^{-n} = \frac{1}{x^n}$.

Developing Skills

In 3–10, write each expression as a rational number without an exponent.

3. 5^{-1} 4. 4^{-2} 5. 6^{-2} 6. $\left(\frac{1}{2}\right)^{-1}$

7. $\left(\frac{1}{5}\right)^{-3}$ 8. $\left(\frac{2}{3}\right)^{-1}$ 9. $\frac{3^0}{4^{-2}}$ 10. $\frac{(2 \cdot 5)^{-4}}{5^{-2}}$

In 11–22, find the value of each expression when $x \neq 0$.

11. 7^0 12. $(-5)^0$ 13. x^0 14. -4^0

15. $(4x)^0$ 16. $4x^0$ 17. $-2x^0$ 18. $(-2x)^0$

19. $\left(\frac{3}{4}\right)^0$ 20. $\frac{3^0}{4}$ 21. $\frac{3^0}{4^0}$ 22. $\frac{3x^0}{(4x)^0}$

In 23–34, evaluate each function for the given value. Be sure to show your work.

23. $f(x) = x^{-3} \cdot x^4$; $f(1)$

24. $f(x) = x + x^{-5}$; $f(3)$

25. $f(x) = (2x)^{-6} \div x^3$; $f(-3)$

26. $f(x) = (x^{-7})^4$; $f(-6)$

27. $f(x) = \left(\frac{1}{x} + \frac{3}{2}\right)^{-2}$; $f(2)$

28. $f(x) = 10^x + 10^{-2x}$; $f(3)$

29. $f(x) = x^{-7} \div x^8$; $f\left(\frac{3}{4}\right)$

30. $f(x) = (3x^{-3} - 2x^{-3})^2$; $f(-2)$

31. $f(x) = x^8\left(x^{-2} + \frac{1}{x^3}\right)$; $f\left(\frac{1}{2}\right)$

32. $f(x) = \left(\frac{x^{-1}}{(2x)^{-2}}\right)^{-1}$; $f(8)$

33. $f(x) = \frac{1}{1 + \frac{2}{x^{-1}}}$; $f(-5)$

34. $f(x) = 4\left(\frac{1}{2}\right)^{-x} + 3\left(\frac{1}{2}\right)^{-x}$; $f(3)$

In 35–63, write each expression with only positive exponents and express the answer in simplest form. The variables are not equal to zero.

35. x^{-4} 36. a^{-6} 37. y^{-5} 38. $2x^{-2}$

39. $7a^{-4}$ 40. $-5y^{-8}$ 41. $(2x)^{-2}$ 42. $(3a)^{-4}$

43. $(4y)^{-3}$ 44. $-(2x)^{-2}$ 45. $-(3a)^{-4}$ 46. $(-2x)^{-2}$

47. $\frac{1}{x^{-3}}$ 48. $\frac{1}{y^{-7}}$ 49. $\frac{3}{a^{-5}}$ 50. $\frac{6}{a^{-4}}$

51. $\frac{9x^2}{a^{-3}}$ 52. $\frac{(-x)^{-5}}{x^{-3}}$ 53. $\frac{(2a)^{-1}}{2(a)^{-2}}$ 54. $\frac{4y^{-3}}{2y^{-1}}$